

Longitudinal Transitions Of Baryon Resonances In Constituent Quark Model

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Abstract

A longitudinal transition operator that satisfies the gauge invariance requirement is introduced in constituent quark model. The corresponding longitudinal transitions between the nucleon and baryon resonances are calculated. We show that the study of the longitudinal coupling plays an important role in understanding the structure of baryons.

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1. Introduction

The electromagnetic transition between the nucleon and excited baryons has been shown to be a very important probe to the structure of nucleon and baryon resonances. A significant progress has been made since the theoretical investigations by Copley, Karl and Obryk[1], and Feynman, Kisslinger and Ravndal[2], who presented first evidence of underlying $SU(6) \otimes O(3)$ symmetry of baryon spectrum. Recent studies have shown[3] that the relativistic effects are required in order to give a consistent description of baryon spectrum and its transitional properties, moreover, they are also essential to generate the model independent results in the low energy limit, such as the low energy theorem in the Compton scattering and corresponding Drell-Hearn-Gerasimov sum rule[4]. The calculations including the relativistic effects in more realistic potential quark models, such as the Isgur-Karl[5] model and its relativised version[6], have shown that the successes of the nonrelativistic quark model have been preserved, thus both spectroscopy and transitions of baryon resonances can be described in the same framework.

However, these studies have mostly concentrated on the transverse transition amplitudes $A_{1/2}$ and $A_{3/2}$, and there is an additional longitudinal transition amplitude $S_{1/2}$ in the electroproduction that has not been systematically studied. Although attempts[7] have been made to investigate the longitudinal transition, there is an important theoretical issue which was not treated consistently in these investigations; the usual expression for the longitudinal transition operator in a quark system

$$H_{em}^L = \epsilon_0 J_0 - \epsilon_3 J_3, \quad (1)$$

where

$$J_0 = \sum_j \sqrt{\frac{2\pi}{k_0}} e_j e^{i\mathbf{k}\cdot\mathbf{r}_j} \quad (2)$$

and

$$J_3 = \sum_j \sqrt{\frac{2\pi}{k_0}} e_j \frac{1}{2m_q} [p_{3,j} e^{i\mathbf{k}\cdot\mathbf{r}_j} + e^{i\mathbf{k}\cdot\mathbf{r}_j} p_{3,j}] \quad (3)$$

and quark j at position \mathbf{r}_j has mass and charge m_j and e_j , does not satisfy the gauge-invariance constraint

$$k_\mu J^\mu = 0, \quad (4)$$

where $k_\mu = \{k_0, 0, 0, k_3\}$ is the momentum of the photon. This was noticed some time ago[10], and was emphasized by Bourdeau and Mukhopadhyay[8]

in their study of the transition $\gamma_v N \rightarrow \Delta$ in the Isgur–Karl[5] and the Vent–Baym–Jackson models[9]. One solution[10] to this problem is to add an *ad hoc* current

$$J'_3 = -\frac{k_3 J_3 - k_0 J_0}{|k_3|^2} k_3 \quad (5)$$

to Eq. 3, which was used in the calculation of the longitudinal transitions between the nucleon and baryon resonances[7].

The focus of this paper is to derive a longitudinal transition operator that satisfies the gauge invariant condition, and use this transition operator to study the longitudinal transitions between the nucleon and baryon resonances. In Ref. [3], we shown that the current conservation in the nonrelativistic limit is equivalent to the energy conservation with the nonrelativistic kinematics. Thus, in addition to the truncated model space problem discussed in Ref. [8], the current conservation will break down due to the nonrelativistic treatment of the recoil effects. In next section, we will show that the relativistic electromagnetic current does satisfy the gauge invariant condition, assuming that the wavefunction is the eigenstate of the relativistic Hamiltonian. Thus, the problem could be avoided by imposing the current conservation in the relativistic limit and then extracting the appropriate nonrelativistic expression, the resulting longitudinal transition operator will be gauge invariant.

In section 3, we calculate the longitudinal transition amplitudes $S_{1/2}$ using the new transition operator derived in section 2. We find significant differences between our results with those in Ref. [7], in which the current in Eq. 5 was added to Eq. 1. We present our results in terms of the ratio between the longitudinal and the transverse cross sections, which would be easier to compare with the experimental data. Finally, the conclusions will be given in section 4.

2. A gauge invariant longitudinal transition operator

The H_{em}^L in Eq. 1 follows from a nonrelativistic approximation to the longitudinal quark–photon vertex,

$$H_{em}^{rel} = \epsilon_0 J_0^{rel} - \epsilon_3 J_3^{rel}, \quad (6)$$

where

$$J_0^{rel} = \sqrt{\frac{2\pi}{k_0}} \sum_{j=1}^3 e_j e^{i\mathbf{k}\cdot\mathbf{r}_j} \quad (7)$$

and

$$J_3^{rel} = \sqrt{\frac{2\pi}{k_0}} \sum_{j=1}^3 e_j \alpha_{3,j} e^{i\mathbf{k}\cdot\mathbf{r}_j}, \quad (8)$$

and $\alpha_{3,j}$ are Dirac matrices. We can rewrite the current J_3 as

$$\begin{aligned} J_3^{rel} &= \sqrt{\frac{4\pi}{2k_0}} \left[\sum_{j=1}^3 \boldsymbol{\alpha}_j \cdot \mathbf{p}_j, \sum_{j=1}^3 e_j e^{i\mathbf{k} \cdot \mathbf{r}_j} \frac{1}{k_3} \right] \\ &= \sqrt{\frac{4\pi}{2k_0}} \left[H_b, \sum_{j=1}^3 e_j e^{i\mathbf{k} \cdot \mathbf{r}_j} \frac{1}{k_3} \right], \end{aligned} \quad (9)$$

where the Hamiltonian[11] in Eq. 2-11 for a three-body system is

$$\begin{aligned} H = \sum_{i=1}^3 \{ \boldsymbol{\alpha}_i \cdot \mathbf{p}_i + \beta_i m_i \} + \sum_{i<j} \left\{ V_v(\mathbf{r})(1 - \frac{1}{2} \boldsymbol{\alpha}_i \cdot \boldsymbol{\alpha}_j) \right. \\ \left. + \frac{1}{2} \boldsymbol{\alpha}_i \cdot \mathbf{r} \boldsymbol{\alpha}_j \cdot \mathbf{r} \frac{V'_v(\mathbf{r})}{|\mathbf{r}|} + \beta_i \beta_j V_s(\mathbf{r}) \right\}, \end{aligned} \quad (10)$$

where $\mathbf{r} = \mathbf{r}_i - \mathbf{r}_j$, $V'_v = \frac{dV_v(\mathbf{r})}{dr}$, and $V_v(\mathbf{r})$ and $V_s(\mathbf{r})$ denote vector and scalar binding potentials for the quark system. Typically, $V_s(\mathbf{r})$ could be a long range scalar linear potential and $V_v(\mathbf{r})$ a single-gluon exchange potential. Thus

$$\begin{aligned} \langle \Psi_f^{rel} | J_3^{rel} | \Psi_i^{rel} \rangle &= \langle \Psi_f^{rel} | \left[H_b, \sum_{j=1}^3 e_j e^{i\mathbf{k} \cdot \mathbf{r}_j} \frac{1}{k_3} \right] | \Psi_i^{rel} \rangle \\ &= (E_f^{rel} - E_i^{rel}) \langle \Psi_f^{rel} | \sum_{j=1}^3 e_j e^{i\mathbf{k} \cdot \mathbf{r}_j} | \Psi_i^{rel} \rangle \frac{1}{k_3} \\ &= \frac{(E_f^{rel} - E_i^{rel})}{k_3} \langle \Psi_f^{rel} | J_0^{rel} | \Psi_i^{rel} \rangle, \end{aligned} \quad (11)$$

where the initial- and final-state wavefunctions $|\Psi_i^{rel}\rangle$ and $|\Psi_f^{rel}\rangle$ must be eigenfunctions of the Hamiltonian H_b . In a radiative transition the energy difference between initial and final states equals the photon energy, that is

$$E_f^{rel} - E_i^{rel} = k_0. \quad (12)$$

Note that Eq. 12 is exact in relativistic limit, so we have the gauge invariance constraint

$$k^\mu J_\mu^{rel} = k_0 J_0 - k_3 J_3 = 0. \quad (13)$$

Since the currents J_0^{rel} and J_3^{rel} have different transformation to the nonrelativistic limit, in particular the nonrelativistic kinematics is used in Eq. 12, the current are no longer conserved in the nonrelativistic limit. Moreover, Eq. 11 shows that the binding potential plays an important role in the current J_3^{rel} ,

thus the truncated model space will further destroy the current conservation, which has been discussed in detail in Ref. [8].

This problem could be avoid if we take a different approach to transform H_{em}^{rel} in Eq. 6 into the nonrelativistic limit; since the current conservation is exact in the relativistic limit, we substitute Eq. 13 into Eq. 6;

$$H_{em}^{rel} = \left[\epsilon_0 - \epsilon_3 \frac{k_0}{k_3} \right] J_0^{rel}, \quad (14)$$

and we chose the longitudinal polarization vector ϵ_μ^L

$$\epsilon_\mu^L = \{\epsilon_0, 0, 0, \epsilon_z\} = \left\{ \frac{k_3}{\sqrt{Q^2}}, 0, 0, \frac{k_0}{\sqrt{Q^2}} \right\}, \quad (15)$$

so the gauge invariant condition,

$$k^\mu \cdot \epsilon_\mu = 0, \quad (16)$$

is satisfied (Q is the virtual photon mass). Note also that

$$\epsilon_0 - \frac{\epsilon_z k_0}{k_z} = \frac{\sqrt{Q^2}}{k_z}, \quad (17)$$

which leads to

$$\langle \Psi_f^{rel} | H_{e,m}^{rel} | \Psi_i^{rel} \rangle = \sum_j \frac{\sqrt{Q^2}}{k_3} \langle \Psi_f^{rel} | J_0^{rel} | \Psi_i^{rel} \rangle. \quad (18)$$

Of course, the longitudinal electromagnetic interaction should be proportional to $\sqrt{Q^2}$, and vanishes in the real photon limit $Q^2 = 0$. This is a direct consequence of the gauge invariance.

The nonrelativistic expansion of Eq. 18 has been given in Ref. [12];

$$\begin{aligned} J_0 = & \sqrt{\frac{2\pi}{k_0}} \left\{ \sum_j \left(e_j + \frac{ie_j}{4m_j^2} \mathbf{k} \cdot (\boldsymbol{\sigma}_j \times \mathbf{p}_j) \right) e^{i\mathbf{k} \cdot \mathbf{r}_j} \right. \\ & \left. - \sum_{j < l} \frac{i}{4M_T} \left(\frac{\boldsymbol{\sigma}_j}{m_j} - \frac{\boldsymbol{\sigma}_l}{m_l} \right) \cdot (e_j \mathbf{k} \times \mathbf{p}_l e^{i\mathbf{k} \cdot \mathbf{r}_j} - e_l \mathbf{k} \times \mathbf{p}_j e^{i\mathbf{k} \cdot \mathbf{r}_l}) \right\}, \quad (19) \end{aligned}$$

where the first term is the charge operator, which is conventionally used in the calculation of longitudinal helicity amplitudes; the second and third terms are spin-orbit and nonadditive terms which have counterparts in the transverse electromagnetic transition[3]. The spin-orbit and nonadditive terms represent

$O(v^2/c^2)$ relativistic corrections to the first term, which have long been known to be necessary even for systems of free particles, if low-energy theorem and Drell-Hearn-Gerasimov sum rule are to be satisfied[13] for the real photon case. The longitudinal helicity amplitude $S_{\frac{1}{2}}$ is defined by

$$S_{\frac{1}{2}} = \langle \Psi_f | J_0 | \Psi_i \rangle \quad (20)$$

where J_0 is given by Eq. 19. The group structure of J_0 is

$$J_0 = A\mathbf{I} + B(S_+L_- - S_-L_+), \quad (21)$$

where \mathbf{I} is the identity operator, A and B are the coefficients determined by Eq. 19. The second term corresponds to the spin-orbit and nonadditive terms, and require that the spin and orbital angular momentum z-component change by ± 1 unit in a transition. Thus, if there is no orbital angular momentum in the initial and final state wavefunctions, the contribution from the second term vanishes. In particular, the selection rule[14] that the longitudinal helicity amplitudes vanish for the transition between the nucleon and hybrid states survives these relativistic corrections if the quark spatial wavefunction of a hybrid state is essentially the same as that of the nucleon and does not have orbital angular momentum.

It should be noted that the expression for H_{em} may not be unique in the nonrelativistic limit; for example, H_{em} can also be written as

$$H_{em} = \frac{\sqrt{Q^2}}{k_0} J_3 \quad (22)$$

due to the current conservation in the relativistic limit. The nonrelativistic expression of J_3 , however, is much more complicated due to the explicit presence of the binding potential shown in Eq. 11, and the problem of the truncated model space becomes important. Moreover, the recoil effects explicitly depend on the choice of the frame, which is also a source of the theoretical uncertainty. This is why Eq. 20 is more convenient and simpler to use, as the explicit dependence of the recoil effects on the choice of frame is eliminated.

3. The Longitudinal Coupling of Baryon Resonances

In Table 1, we show the analytical expressions of the longitudinal transition between the nucleon and the baryon resonances in the $SU(6) \otimes O(3)$ symmetry limit. The terms proportional to $\frac{\alpha^2}{m_q^2}$ represent the relativistic contributions that come from the spin-orbit and nonadditive term in Eq. 19. The relativistic

effects only scale the longitudinal coupling amplitudes, and do not affect the general behaviour of Q^2 dependence of $S_{1/2}(Q^2)$. Therefore, the ratio between the longitudinal couplings of the resonances $S_{11}(1530)$ and $D_{13}(1520)$ would be independent of Q^2 since their masses are approximately equal. This ratio is determined by the Clebsch-Gordon coefficients in the nonrelativistic limit;

$$\frac{S_{1/2}(S_{11}(1530))}{S_{1/2}(D_{13}(1520))} = -\frac{1}{\sqrt{2}}, \quad (23)$$

and the relativistic effects would change this ratio by a factor of $\frac{1+\frac{\alpha^2}{6m_q^2}}{1-\frac{\alpha^2}{12m_q^2}}$. The standard quark model parameters $m_q = 0.33$ GeV and $\alpha^2 = 0.17$ GeV² give

$$\frac{1+\frac{\alpha^2}{6m_q^2}}{1-\frac{\alpha^2}{12m_q^2}} = 1.45, \quad (24)$$

thus, this give an approximately -1 ratio with the relativistic corrections. The relativistic effects also lead to a nonzero longitudinal transitions between the resonance $D_{15}(1670)$, which vanishes for the nonrelativistic transition operator. This gives us an important experimental test for the spin-orbit and nonadditive term in the longitudinal transition operator.

The calculation of the Q^2 dependence of $S_{1/2}(Q^2)$ follows the procedure of Foster and Hughes[15]; a Lorentz boost factor in the spatial integrals are introduced so that

$$R(k) \rightarrow \frac{1}{\gamma^2} R\left(\frac{k}{\gamma}\right), \quad (25)$$

where the Lorentz boost factor γ can be written as

$$\gamma^2 = 1 + \frac{k^2}{(M_r + M_p)^2} \quad (26)$$

in the equal velocity frame and

$$k^2(EVF) = \frac{(M_r^2 - M_p^2)^2}{2M_r M_p} + \frac{Q^2(M_r^2 + M_p^2)}{4M_r M_p} \quad (27)$$

for the initial nucleon mass M_p and final resonance mass M_r . The corresponding Q^2 dependence of $S_{1/2}(Q^2)$ for $S_{11}(1530)$ is given by

$$S_{1/2}(Q^2) = \frac{2}{3} \sqrt{\frac{\pi}{k_0}} \mu m_q \frac{k}{\gamma^3 \alpha} \left(1 + \frac{\alpha^2}{6m_q^2}\right) \frac{1}{1 + Q^2/0.8} e^{-\frac{k^2}{6\alpha^2 \gamma^2}}, \quad (28)$$

where an *ad hoc* form factor $\frac{1}{1+Q^2/0.8}$ is being added, since it gives a better quantitative description of the Q^2 dependence of transverse helicity amplitudes[15] for $\alpha^2 = 0.17 \text{ GeV}^2$, and it becomes unnecessary with $\alpha^2 = 0.09 \text{ GeV}^2$ [3]. In Fig.1, we show the Q^2 dependence of the longitudinal amplitude $S_{1/2}(Q^2)$ for the resonance $S_{11}(1530)$ in the $SU(6) \otimes O(3)$ symmetry limit. The relativistic effects increase $S_{1/2}(Q^2)$ by about 25 percent. The resulting $S_{1/2}(Q^2)$ is significantly larger than that in Ref. [7], and in better agreement with the analysis by Gerhardt[16], who extracted the longitudinal transition amplitudes from the electroproduction data. This shows the importance of choosing the correct transition operator for the longitudinal coupling.

A more important quantity is the ratio between the longitudinal coupling and transverse coupling amplitudes,

$$R = \frac{S_{1/2}^2(Q^2)}{A_{1/2}^2(Q^2) + A_{3/2}^2(Q^2)}, \quad (29)$$

in which the common factors, such as the *ad hoc* form factor in Eq. 28, may cancel out, thus provides us a direct probe of the underlying structure of the resonance. The analytic expressions for transverse helicity amplitudes $A_{1/2}$ and $A_{3/2}$ are given in Ref. [3]. The Q^2 dependence of this ratio for the resonance $S_{11}(1530)$ is shown in Fig. 2, it shows a strong presence of the longitudinal transitions for this resonance. Moreover, the result for the transverse helicity amplitude $A_{1/2}(Q^2)$ [3] in the symmetry limit is twice larger than the experimental data and it decreases too fast as Q^2 increases, this indicates a strong configuration mixing for the resonance $S_{11}(1530)$. The data of the Q^2 dependence of this ratio provide another crucial test to the various potential quark models with different binding potentials. The extension of this investigation to include the configuration mixing is in progress. It is interesting to note that the configuration mixings in the Isgur-Karl model[3] do not change the result of naive quark model significantly due to the strong presence of the 70 multiplet state in the nucleon wavefunction.

It is straightforward to obtain the Q^2 dependence of the longitudinal transition for the resonance $D_{13}(1520)$, which Eq. 24 shows that its longitudinal transition approximately equals to that of the resonance $S_{11}(1520)$ with a opposite sign. Thus, strong contributions from the longitudinal transitions are expected for the P-wave resonances, and furthermore, the relativistic effects contributes significantly to $S_{1/2}(Q^2)$ because of the nonzero orbital angular momentum in the wavefunction of P-wave resonances.

In Fig. 3, we show the Q^2 dependence of $S_{1/2}(Q^2)$ for the resonance $F_{16}(1688)$, the relativistic effects reduces the longitudinal transitions significantly, which are in better agreement with the result of Gerhardt[16]. The Q^2

dependence of the longitudinal and transverse transitions for this resonance is shown in Fig. 4; the longitudinal transitions are much smaller in this case, in particular, it is less than 0.1 with the relativistic corrections. The naive quark model[1, 17] does give a good description of the transverse helicity amplitudes even quantitatively. More precise data for the longitudinal transition would provide us more insights into the structure of this resonance; deviations from the prediction in Figs. 3 and 4 would be evidences for the configuration mixings.

The longitudinal transition between the resonance $P_{33}(1232)$ and the nucleon vanishes in the symmetry limit. However, if there is a small component of the orbital angular momentum in both wavefunctions of the nucleon and the resonance $P_{33}(1232)$, the spin-orbit and the nonadditive term in the longitudinal transition operator would lead to a nonzero longitudinal transition between these two states. In Fig. 5, we show the ratio

$$R = -\frac{S_{1/2}(Q^2)}{\sqrt{2}M1} = \frac{\sqrt{2}S_{1/2}(Q^2)}{\sqrt{3}A_{3/2}(Q^2) + A_{1/2}(Q^2)} \quad (30)$$

(notice $S_{1/2}(Q^2)$ in Eq. 20 differs by a factor of $-\frac{1}{\sqrt{2}}$ from Ref. [8]) for the Isgur-Karl model, whose wavefunctions for the nucleon and the resonance $P_{33}(1232)$ are obtained from Ref. [18], and the transverse helicity amplitudes $A_{3/2}(Q^2)$ and $A_{1/2}(Q^2)$ are given in Ref. [3]. We find that the relativistic corrections approximately double this ratio. While the experimental data[19] are inconclusive for a finite ratio $E1/M1$, they do suggest a finite and negative ratio $S_{1/2}/M1$. This suggests that the longitudinal transitions between the nucleon and the resonance $P_{33}(1232)$ might be a better probe to the orbital angular momentum in the nucleon wavefunction than the $E1$ transition, which certainly deserves more attention.

4. Conclusions

We have derived a gauge independent longitudinal transitions operator, in which the relativistic effects are included. The calculations of longitudinal transitions between the nucleon and baryon resonances are made in the symmetry limit. We show that the relativistic effects in the transition operator give important contributions to the longitudinal helicity amplitudes, especially in the transition between the nucleon and the resonance $P_{33}(1232)$.

We show that the longitudinal transitions of baryon resonances play an important role in understanding the structure of baryons. The longitudinal transition amplitude $S_{1/2}(Q^2)$ decreases as the Q^2 increases. Thus, it is important in the small Q^2 regions, in particular, for the transitions between the

nucleon and the P-wave resonance, which is accesible to the experiments at CEBAF.

An extension of this investigation is to study the spin structure function $g_{1,2}(x, Q^2)$ in the resonance region, where the studies[20] have shown that the Q^2 dependence of the spin-structure function is very significant, and the longitudinal transitions amplitudes provide important contributions to the spin-structure functions in the low Q^2 region.

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Figure Caption

1. The Q^2 dependence of $S_{1/2}(Q^2)$ for the resonance $S_{11}(1535)$, the solid line represents the nonrelativistic result and the dashed line includes the relativistic corrections.
2. The ratio between the longitudinal and transverse cross sections for the resonance $S_{11}(1535)$. The solid line represents the nonrelativistic result, and the dashed line includes the relativistic corrections.
3. The same as Fig. 1 for the resonance $F_{15}(1688)$.
4. The same as Fig. 2 for the resonance $F_{15}(1688)$.
5. The ratio between the longitudinal and $M1$ transitions for the resonance $P_{33}(1232)$ in the Isgur-Karl model, see text.

Table 1: Transition matrix elements between the nucleon and baryon resonances in the $SU(6) \otimes O(3)$ symmetry limit. The full matrix elements are obtained by multiplying the entries in this table by a factor $\sqrt{\frac{2\pi}{k_0}} 2\mu m_q e^{-\frac{\mathbf{k}^2}{6\alpha^2}}$, and $S_{\frac{1}{2}}^n = S_{\frac{1}{2}}^p$ for Δ states.

Multiplet	States	Proton	Neutron
[70, 1 ⁻] ₁	$N(^2P_M)\frac{1}{2}^{-1}$	$\frac{1}{3\sqrt{2}} \frac{ \mathbf{k} }{\alpha} \left(1 + \frac{\alpha^2}{6m_q^2}\right)$	$-\frac{1}{3\sqrt{2}} \frac{ \mathbf{k} }{\alpha} \left(1 + \frac{\alpha^2}{6m_q^2}\right)$
	$N(^2P_M)\frac{3}{2}^{-1}$	$-\frac{1}{3} \frac{ \mathbf{k} }{\alpha} \left(1 - \frac{\alpha^2}{12m_q^2}\right)$	$\frac{1}{3} \frac{ \mathbf{k} }{\alpha} \left(1 - \frac{\alpha^2}{12m_q^2}\right)$
	$N(^4P_M)\frac{1}{2}^{-1}$	$\frac{1}{36\sqrt{2}} \frac{\alpha \mathbf{k} }{m_q^2}$	$-\frac{1}{108\sqrt{2}} \frac{\alpha \mathbf{k} }{m_q^2}$
	$N(^4P_M)\frac{3}{2}^{-1}$	$\frac{1}{9\sqrt{10}} \frac{\alpha \mathbf{k} }{m_q^2}$	$-\frac{5}{27\sqrt{10}} \frac{\alpha \mathbf{k} }{m_q^2}$
	$N(^4P_M)\frac{5}{2}^{-1}$	$\frac{1}{12\sqrt{10}} \frac{\alpha \mathbf{k} }{m_q^2}$	$-\frac{5}{36\sqrt{10}} \frac{\alpha \mathbf{k} }{m_q^2}$
	$\Delta(^2P_M)\frac{1}{2}^{-1}$	$-\frac{1}{3\sqrt{2}} \frac{ \mathbf{k} }{\alpha} \left(1 - \frac{\alpha^2}{6m_q^2}\right)$	
	$\Delta(^2P_M)\frac{3}{2}^{-1}$	$\frac{1}{3} \frac{ \mathbf{k} }{\alpha} \left(1 + \frac{\alpha^2}{12m_q^2}\right)$	
[56, 0 ⁺] ₂	$N(^2S_{S'})\frac{1}{2}^{+}$	$-\frac{1}{3\sqrt{6}} \frac{\mathbf{k}^2}{\alpha^2}$	0
	$\Delta(^4S_{S'})\frac{3}{2}^{+}$	0	
[56, 2 ⁺] ₂	$N(^2D_S)\frac{3}{2}^{+}$	$-\frac{1}{3\sqrt{15}} \frac{\mathbf{k}^2}{\alpha^2} \left(1 + \frac{\alpha^2}{2m_q^2}\right)$	$-\frac{\mathbf{k}^2}{12\sqrt{15}m_q^2}$
	$N(^2D_S)\frac{5}{2}^{+}$	$-\frac{1}{3\sqrt{10}} \frac{\mathbf{k}^2}{\alpha^2} \left(1 - \frac{\alpha^2}{3m_q^2}\right)$	$\frac{\mathbf{k}^2}{9\sqrt{10}m_q^2}$
	$\Delta(^4D_S)\frac{1}{2}^{+}$	$-\frac{5\mathbf{k}^2}{72\sqrt{15}m_q^2}$	
	$\Delta(^4D_S)\frac{3}{2}^{+}$	0	
	$\Delta(^4D_S)\frac{5}{2}^{+}$	$\frac{5\sqrt{5}\mathbf{k}^2}{216\sqrt{7}m_q^2}$	
	$\Delta(^4D_S)\frac{7}{2}^{+}$	$\frac{5\mathbf{k}^2}{36\sqrt{105}m_q^2}$	
[70, 0 ⁺] ₂	$N(^2S_{M'})\frac{1}{2}^{+}$	$\frac{1}{18} \frac{\mathbf{k}^2}{\alpha^2}$	$-\frac{1}{18} \frac{\mathbf{k}^2}{\alpha^2}$